On sets meeting every line in a set of measure 1

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One can ask whether there exist a subset S of the plane having a prescribed size on each line, for example it is known that there is a set containing exactly two points from each line.

We heard the following similar question from A. Kumar (but others had asked it before): Does there exist a set on the plane meeting each line in a (λ^1-measurable) set of measure 1? Under CH it is not difficult to construct such a set. On the other hand it is proved (in ZFC) that a set of this type cannot be λ^2-measurable [1]. It remained open whether such a set always exists, or its non-existence is consistent with ZFC.

Using the theory of harmonic functions, and a consistent inequality between two cardinal invariants of the null ideal we proved that consistently, there is no set on the plane intersecting each line in a set of (1-dimensional) measure 1. This is a joint work with Márton Elekes and Zoltán Vidnyánszky.

References

[1] M.N. Kolountzakis, M. Papadimitrakis: *Measurable Steinhaus sets do not exist for finite sets or the integers in the plane.* Bulletin London Math. Soc., 49, 5 (2017), 798-805.